

7.4 Double Angle Identities

LT44: I can use the double angle and half-angle identities to find exact values and prove other identities.

Review

Use half-angle identities to solve $\sin 112.5^\circ$

$$\begin{aligned}\sin\left(\frac{225}{2}\right) &= \sqrt{\frac{1 - \cos 225}{2}} \\ &= \sqrt{\frac{(1 - \frac{\sqrt{2}}{2})^2}{(2)2}} \\ &= \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}\end{aligned}$$

Example

$\sin \theta = \frac{4}{5}$. Terminal side in Quadrant I. Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) \\ &= \left(\frac{24}{25}\right)\end{aligned}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\begin{aligned}\left(\frac{4}{5}\right)^2 + \cos^2 \theta &= 1 \\ -\frac{16}{25} & \quad -\frac{16}{25} \\ \sqrt{\cos^2 \theta} &= \sqrt{\frac{9}{25}} \\ \cos \theta &= \frac{3}{5}\end{aligned}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\begin{aligned}&= \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2} \\ & \quad \rightarrow \frac{16}{9}\end{aligned}$$

$$\begin{aligned}&= \frac{\frac{4}{5}}{\frac{3}{5}} \rightarrow \frac{4}{5} \div \frac{3}{5} \\ &= \frac{4}{3} \quad \frac{4}{5} \cdot \frac{5}{3}\end{aligned}$$

$$= \frac{\frac{8}{3}}{-\frac{7}{9}}$$

$$= \frac{8}{3} \cdot \frac{-9}{7}$$

$$= \left(\frac{-24}{7}\right)$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$= 1 - 2\left(\frac{4}{5}\right)^2$$

$$= 1 - 2\left(\frac{16}{25}\right)$$

$$= 1 - \frac{32}{25}$$

$$= \left(\frac{-7}{25}\right)$$