7.4 Double Angle Identities

LT44: I can use the double angle and half-angle identities to find exact values and prove other identities.

Review

Use half-angle identities to solve $\sin 112.5°$

$$\sin\left(\frac{225}{2}\right) = \sqrt{\frac{1-\cos 225}{2}}$$

$$= \sqrt{\frac{\left(1-\left(-\frac{\sqrt{2}}{2}\right)\right)2}{(2)2}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

Example

$\sin\theta = \frac{4}{5}$. Terminal side in Quadrant I. Find $\sin 2\theta, \cos 2\theta, \tan 2\theta$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{4}{5}\right)^2 + \cos^2\theta = 1$ $\quad -\frac{16}{25}$

$-\frac{16}{25}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{9}{25}}$

$\cos\theta = \frac{3}{5}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}$$

$\to \frac{16}{9}$

$$= \frac{\frac{8}{3}}{-\frac{7}{9}}$$

$$= \frac{8}{3}\cdot\frac{-3}{7}$$

$$= \frac{-24}{7}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{4}{5}}{\frac{3}{5}}$ $\to$ $\frac{4}{5} \div \frac{3}{5}$

$\frac{4}{5}\cdot\frac{5}{3}$

$= \frac{4}{3}$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$= 1 - 2\left(\frac{4}{5}\right)^2$$

$$= 1 - 2\left(\frac{16}{25}\right)$$

$$= 1 - \frac{32}{25}$$

$$= \frac{-7}{25}$$